

of one-third the transverse displacements, but the high frequency content (compared to that in the postcollapse case) is negligible.

Chaos-like behavior was observed in the Dshell finite element model in postcollapse. The broadbanded frequency spectra of the oscillations and the very irregular, but bounded, nodal trajectories in phase space (not shown) are consistent with chaos.

A time step that is too small increases the number of calculations unnecessarily and hastens the accumulation of numerical error.

The deformed geometry plots give an immediate, though qualitative, indication of numerical instability whereas observation of the displacement of a single DOF vs time may not. In cases of numerical instability, Dshell is usually able to converge to a solution that satisfies compatibility and potential energy constraints although arriving at an unrealistic and incorrect deformed geometry. Unbounded trajectories in phase space also indicate numerical instability and give quantitative information as to the onset.

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# Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one Journal page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.

## Comment on "Skin Friction and Velocity Profile Family for Compressible Turbulent Boundary Layers"

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**I** NOTICED the following misprints in the recent article "Skin Friction and Velocity Profile Family for Compressible Turbulent Boundary Layers":<sup>1</sup>

1) In the second line after Eq. (8),  $W(\eta) = 2 \sin^2(\eta\pi/2)$  should read  $W(\eta) = 2 \sin^2(\eta\pi/2)$ .

2) Equation (10), if taken from Ref. 6, should read

$$\Pi = 0.55[1 - \exp(-0.243z_1^{0.5} - 0.298z_1)]$$

where  $z_1 = Re_\theta/425 - 1$ .

3) The horizontal axis in Fig. 1a is  $Re_\theta \times 10^{-3}$ , but in step 2 of the skin-friction algorithm it is said that "calculate  $Re_{\delta_2} = \rho_e u_e \theta / \mu_w$  and find  $\Pi$  from Fig. 1a." Don't you think  $Re_{\delta_2}$  should be replaced by  $Re_\theta$ ?

In addition I would like to make the following comments:

1) The title of the paper implies that the method is applicable to all zero pressure gradient compressible flows from subsonic to hypersonic flows. In the paper the method is tested mainly against hypersonic data; therefore, I think the term "compressible" should have been replaced with "hypersonic."

2) The last line in the skin-friction algorithm states that steps 1-7 are repeated until the solution converges. I am not quite sure how we can be certain of having convergence by specifying only one boundary-layer parameter ( $\Theta$  or  $\delta^*$ ) to predict skin friction and velocity profile. More explanations were needed.

3) In step 4 of the skin-friction algorithm, the definition of  $U_\delta^+$  in relation of Eq. (6) is not clear. The addition of a clear list of notations would have been helpful.

## Reference

- <sup>1</sup>Huang, P. G., Bradshaw, P., and Coakley, T. J., "Skin Friction and Velocity Profile Family for Compressible Turbulent Boundary Layers," *AIAA Journal*, Vol. 31, No. 9, 1993, pp. 1600-1604.

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